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USING HUGHES' SALVO MODEL TO EXAMINE SHIP CHARACTERISTICS IN SURFACE WARFARE

by

Kevin G. Haug

September 2004

Thesis Advisor:	Tom Lucas
Second Reader:	Wayne Hughes

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As resources constrain investment decisions, what combination of parameters most effectively cause one force to defeat another? Using Hughes' Salvo equations, simulations are conducted to investigate the singular and pairwise effects of providing one force an advantage in its offensive power, defensive power, staying power, force size, and information. The purpose is to identify specific combinations that present potential priorities in ship design and force planning. Cases are examined in terms of fraction of forces killed and surviving, and consolidated in a comparison of fractional exchange ratios between the forces. Over the range of parameters explored, when forces are closely matched, a defensive advantage allows a force to outlast another, execute damage, and limit damage incurred to its own force. The Polya distribution of shots shows that the bonus gained by attaining perfect information is a significant edge, and the hazard of failing to deny the enemy the same.

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USING HUGHES' SALVO MODEL TO EXAMINE SHIP CHARACTERISTICS IN SURFACE WARFARE

Kevin G. Haug
Lieutenant, United States Navy
B.A., William Jewell College, 1994

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September 2004

Author: Kevin G. Haug

Approved by: Tom Lucas
Thesis Advisor

Wayne Hughes
Second Reader

James N. Eagle
Chairman, Department of Operations Research

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ABSTRACT

As resources constrain investment decisions, what combination of parameters most effectively cause one force to defeat another? Using Hughes' Salvo equations, simulations are conducted to investigate the singular and pairwise effects of providing one force an advantage in its offensive power, defensive power, staying power, force size, and information. The purpose is to identify specific combinations that present potential priorities in ship design and force planning. Cases are examined in terms of fraction of forces killed and surviving, and consolidated in a comparison of fractional exchange ratios between the forces. Over the range of parameters explored, when forces are closely matched, a defensive advantage allows a force to outlast another, execute damage, and limit damage incurred to its own force. The Polya distribution of shots shows that the bonus gained by attaining perfect information is a significant edge, and the hazard of failing to deny the enemy the same.

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THESIS DISCLAIMER

The reader is cautioned that the computer programs presented in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logical errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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EXECUTIVE SUMMARY

Naval planners must make decisions allocating resources for shipbuilding to obtain the most capable forces. But, what characteristics provide the most capable force? A few primary factors that affect the outcome of naval surface combat include the number of missiles a ship can fire (offensive power), the number of missiles it can defeat (defensive power), the amount of damage it can sustain (staying power), and situational awareness (information). Of course, the size of the force is also a major factor. Using a stochastic simulation based on Hughes' Salvo Model, one side is given two advantages to examine how their interactions affect the outcome. Using this model, shots are distributed to the opponent force using a uniform random distribution. This forms the basis for the second part of the study. Does changing the allocation of how shots are distributed change the outcome of the simulation?

Although we can gain insights through exploratory analysis using modeling and simulation, it is important to avoid making too broad of generalizations. This study covers a small range of data in the variables explored, significant changes in values outside the range explored is likely to yield very different conclusions. Additionally, this model has not been validated, so conclusions derived are based on assumptions made in the model, which may or may not be valid.

First, blue forces are given a one unit increase in the values of offensive, defensive, and staying powers, an information advantage, and a one unit increase in the number of units in its force. The information advantage is defined as the ability of blue forces to obtain perfect battle damage assessment of the outcome of its previous salvo. This allows blue to prevent wasting shots on units that are already out of action. Conclusions from the single parameter advantages show the following:

1. An increase in force strength or firepower yields a larger portion of red killed with a lesser effect of more blue units surviving.
2. An increase in defensive power yields a larger portion of blue surviving with a lesser effect of more red units killed.
3. An increase in staying power yields a larger portion of blue surviving with a lesser effect of more red units killed, but to a lesser degree than increasing defensive power.
4. An increase in information yields a larger portion of red killed in combat where there are large numbers of offensive missiles fired, but otherwise has little effect.

Next, blue forces are given a one unit increase in two of the parameters to examine the interactions. Conclusions from the pairwise interactions show the following:

1. Information, i.e., perfect battle damage assessment, contributes less to the outcomes than any single parameter.
2. A purely defensive focus of staying power and defensive power will allow blue to fight the battle with minor losses, but blue will be less successful in eliminating red forces.
3. A purely offensive focus of force and firepower allows blue to eliminate virtually all of red's forces, but blue stands a risk of losing his force in the process.
4. The remaining combinations have varying degrees of success in the amount of red eliminated and blue surviving and selection should be determined based on which is preferred.

To summarize the effects, Table 1 provides the fractional exchange ratios, that is the fraction of blue forces killed to the fraction of red forces killed. Each ratio is the result of 120 different combinations of parameters, each of which is simulated with 1000 replications. The values can be interpreted as the number of blue units a force can expect to lose for a single red kill.

Blue Advantages	Fractional Exchange Ratio
Force	0.42
Information	0.78
Firepower	0.42
Staying Power	0.35
Defensive Power	0.2
Firepower Defensive Power	0.12
Force Defensive Power	0.06
Force Firepower	0.26
Force Information	0.35
Information Defensive Power	0.17
Information Firepower	0.4
Staying Power Information	0.26
Staying Power Force	0.24
Staying Power Defensive Power	0.08
Staying Power Firepower	0.13

Table 1. Fractional exchange ratios for combinations for the ten cases examined

The second portion of the thesis examines how applying Hughes' Salvo Model with a non-uniform distribution of shots impacts results of the simulation. Historical data has shown that units that perform better in combat continue to do so and are attributed with a higher number of kills than the average unit. If the same distribution applies to targeted units, then a Polya distribution of shots is an accurate selection. Under the Polya distribution, a unit that has been previously targeted is more likely to be targeted on the next engagement. This may be desired if a unit in the opponent force is a command and control ship, or perhaps if the closest ship is the largest threat. The first exploration using the Polya distribution of shots is compared to the uniform distribution of shots to compare the differences. The Polya distribution results in higher lethality combat, in that between thirty to seventy percent of the forces can expect to be destroyed on average. However, as a function of the distribution, it is not possible that on average a force can be completely eliminated. To account for this, blue is again given an information advantage to allow him to recover from wasting shots on the highest probability target, the one that is already out of action. The results show that for the most frequently occurring cases, blue is able to kill more than eighty percent of the red forces. Thus, the importance of the information is significant. However, what if red is also given this information advantage? The outcome is a much higher lethality for both forces. For over sixty percent of the cases observed, more than seventy percent of the forces are killed. This stresses that while gaining the information advantage, it is at least as important to deny the opponent that same information, or both forces are more likely to be destroyed.

I. MODELING TO DETERMINE SHIP DESIGN

A. THE PROBLEM

Naval planners must make decisions allocating resources for shipbuilding to obtain the most capable forces. What characteristics determine the most capable force? Are there situations in which a superior force should not engage because it faces a significant risk of suffering unacceptable losses by the inferior force? Combat modeling can give insights to answering questions like these.

Some primary factors that affect naval battles include a ship's firepower, speed, endurance, armor, and the size of its force. Speed is a point of contention. Although increases in speed have been shown to project a strategic advantage, questions arise in speed's significance in a tactical environment in which the threat faced is a supersonic missile. With the advent of modern missiles and their counters, a defensive ability is added as a contributing factor, as is information on the opponent forces. Hughes' Salvo Model considers the parameters of firepower, staying power, defensive power, and force size as the primary drivers in exploring battle outcomes in an elegantly simple model (Hughes, 1992). Exploration has shown that some basic configurations, such as concentrating too much on firepower at the expensive staying power, should be avoided. There is little worth in a ship that can exterminate an entire enemy force if it can also be struck down in the first round of combat.

McGunnigle extended Hughes' deterministic model into the Stochastic Salvo Model. This allowed him to examine the effects of the scouting, intelligence, and damage assessment to determine the importance of information (McGunnigle, 1999). McGunnigle simulates homogenous forces, stochastically determining the number of shots fired and defeated based on a unit's status. He assumes a random uniform distribution of shots to targeted ships. McGunnigle assumes an information advantage in which the blue forces know the capabilities, status, and exact positions of enemy forces and allocate their weapons accordingly. This information advantage allows blue to overwhelm individual red units efficiently, but cease to fire at a red unit if it is put out of action. His analysis shows that although information can contribute to an increased success for blue units, a force advantage (i.e., more ships) provides a more certain likelihood of success for blue.

Combat models are used to attempt to gain insight into how varying combat parameters might affect the outcome of a battle. The challenge is to develop a model that incorporates sufficient variables to capture the essence of a battle in a simple form—exactly how simple is too simple is a point of debate (Lucas and McGunnigle, 2002). While it is possible to attempt to model the physics and stochastic elements of a major naval surface battle, doing so is perhaps not always the best course of action. A simulation such as McGunnigle's Stochastic Salvo Model allows great flexibility in changing parameters, exposing events that may seem trivial, but in fact drive the outcome of the simulation. A problem with high resolution models is the significant run time for a single simulation. To produce a result with a measure that contains a measure of variability requires many runs. A point estimate of the battle may be determined in a few hours, but to see how variable that estimate is can take days, weeks, even months, thus hindering their use as exploratory tools. Furthermore, generating input data to feed large models may take many months.

The alternative is a model that has relatively few parameters. This type of model aggregates a wide range of variables into a single parameter in an attempt to approximate the essential dynamics of the situation. Which parameters are selected is critical, as failure to incorporate a driver in the combat simulation will yield results that will be challenged as invalid for failing to incorporate that vital missing element. However, the speed of such a model is orders of magnitude faster than the high resolution models, allowing for rapid repetitions. Thus, if one believes the values of the parameters are appropriate and capture the combat drivers, then the measure can be estimated with a good understanding of its variability. A second benefit of such models is that they may be simple enough to be solved analytically, allowing for verification of a developed simulation.

For naval surface combat, such an alternative model exists, the Hughes Salvo Model. Developed by Wayne Hughes, this model uses relatively few parameters and has been tested using data from modern naval battles involving missile combat. The Salvo Model has been extended to include a Stochastic Salvo Model variant and was used to explore the value of information in combat (McGunnigle, 1999). This thesis extends the Stochastic Salvo Model implementation by McGunnigle to explore parameters in naval combat to address the following questions:

1. What is the effect of interactions between ship characteristics?
2. What is the effect if the shot assignment distribution is non-uniform?

B. THE CONTRIBUTION

The single effects of numerical advantage and information advantage are explored in McGunnigle's thesis and summarized in Chapter III. This thesis continues that work by additionally examining superior staying power, firepower advantage, and defensive power advantages. Are there combinations (synergistic effects) of these factors that may lead to enhanced performance? Or will the model expose a combination that appears more capable, but in combat produces unstable results—similar to Hughes' point on the balance between offensive and staying power that must be incorporated into ship design?

The Salvo Models make the assumption of a uniform distribution of shots between forces. That is, the shots in a salvo are spread evenly over the opponent's force. From an operational standpoint this may not be the case. If the opponent possesses a more capable ship, or a command-and-control ship, it is reasonable to assume that ship would be targeted first in an attempt to reduce the combat effectiveness of the opponent force. If there are merchant ships between the forces, one opponent ship likely presents a better targeting opportunity than the other units in the opposing force. There is evidence that shooters who have success continue to have success, and conversely those who struggle continue to do so (Bolmarcich, 2000). This can be explained by superior unit training, a better maintained or higher performing weapon or targeting system, or simply a higher performing set of individuals. The Polya distribution captures the effects of such a phenomenon and is used to assess whether the sensitivity of the uniform distribution assumption is significant.

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II. BUILDING THE MODEL

A. INTRODUCTION

This chapter explains the basic Hughes' Salvo Model, a small excerpt of Armstong's work looking at stochastic effects, and the development of McGunnigle's simulation for analyzing combat by implementing a Stochastic Salvo Model. Using Hughes' Salvo Model, and the McGunnigle's formulation, a Java simulation is created to explore the singular and pairwise effects of firepower, defensive power, staying power, force size and information advantages, as well as the exploration of a non-uniform allocation of shots.

B. THE BASIC MODEL

The basic Hughes' Salvo Model examines the fraction of forces remaining after a salvo of shots are fired and defended by each force, taken directly from Hughes (1992).

$$\Delta B = \frac{\alpha A - b_3 B}{b_1}, \Delta A = \frac{\beta B - a_3 A}{a_1}.$$

A = number of units in force A

B = number of units in force B

α = number of well-aimed missiles fired by each A unit

β = number of well-aimed missiles fired by each B unit

a_1 = number of hits by B's missiles needed to put one A out of action

b_1 = number of hits by A's missiles needed to put one B out of action

a_3 = number of well-aimed missiles destroyed by each A

b_3 = number of well-aimed missiles destroyed by each B

ΔA = number of units in force A out of action from B's salvo

ΔB = number of units in force B out of action from A's salvo

The αA term represents the number of well-aimed shots by force A in a salvo. The $b_3 B$ term represents the number of well-aimed shots that are defeated by the defensive systems of force B. The difference of these terms is the number of well-aimed shots by force A from which force B will incur damage. Dividing this by the number of shots required to put a unit of B out of action results in ΔB, the number of units put out of action in a salvo from force A. Force B's well-aimed shots are computed similarly to determine the number of units of Force A that are put out of action.

Hughes' Salvo Model makes the following assumptions:

1. The striking power of the attacker is equal to the sum of the good shots launched by the ships of a force.
2. Good shots are uniformly distributed over the opponent's force.
3. Good shots will be engaged by defensive forces until the point of saturation is achieved, at which point all good shots will hit.
4. Hits on a force will decrease a unit's effectiveness linearly until it is out of action.
5. A ship's staying power is the number of shots required to achieve a mission kill, not destroy the ship.
6. All units in one force can identify and engage all units in the other force.
7. Losses are measured in terms of units put out of action.
8. The key attributes in determining outcomes include striking power, staying power, defensive power, scouting effectiveness, soft-kill counteractions, defensive readiness, and training.
9. Enemy capabilities cannot be exactly known, and thus the results must be based on an average of possibilities.
10. Command and control are not explicitly considered as they are assumed to be incorporated into the parameters of the individual ships.

Some important findings of Hughes include the following:

1. Staying power is a vital parameter in that neglect can lead to unstable results, and it should be balanced carefully with a units' offensive power.
2. The most significant factor is a force size advantage.

3. Scouting is important, particularly in a case in which there is an imbalance associated with high firepower and low defensive and staying power.
4. A force that can deliver an uncountered first strike will often gain a significant advantage, even if the attacked force is highly superior.

Armstrong (2003) explores Hughes' Salvo Model analytically to gain insight into how the outcome can be determined, based on the lethality level of combat for like forces. He defines lethality in three terms, low, moderate, and high. In the low lethality situation, the ratio of the blue force offensive power to the red force defensive power is less than the ratio of the blue force defensive power to the red force offensive power. More simply, the blue force is constructed as a force more concerned with self-preservation than with the destruction of the enemy—and vice versa. There exists an abrupt breakpoint in low lethality combat at which a force is either completely eliminated and the other is undamaged, or neither side can damage the opponent. The primary factor in determining the outcome of low-level combat is the number of ships required to push the size of force over this threshold in homogenous cases.

The moderate lethality situation occurs when one side has sufficient offensive power to overpower the opponent's defenses to inflict damage, but not enough to guarantee that the unit is put out of action in a single salvo. In the moderate lethality equations, there are five outcomes, total destruction of the opponent for a force, a stalemate, or a case where one force destroys the opponent while sustaining damage itself. Where exactly the outcome falls depends on the parameters of each force's ships. The high lethality situation occurs when a force has sufficient offensive power to destroy the opponent completely in a single salvo. This is the imbalance Hughes warns against. Armstrong analytically shows that as the ratio of blue's offensive power to its defensive power and its ability to sustain damage decreases below the ratio of number of blue ships to the number of red ships, red will prevail although it will suffer some damage. If the unbalance becomes greater, when the ratio of blue's offensive power to red's defensive power is less than the ratio of the number of red ships to the number of blue ships, then red prevails while suffering no damage to its own forces.

The following procedure, adopted from McGunnigle's (1999) work is applied to determine the outcome from a single salvo.

1. Determine the number of shots each unit in each force is capable of firing.
2. Determine the number of good shots each unit is capable of firing. A good shot is determined stochastically.
3. Randomly assign the good shots of each force to the opposing force. McGunnigle assigned shots using a uniform distribution.
4. For each ship in each force, determine the number of good shots a unit is capable of defeating.
5. For each unit in each force, determine the number of good shots that have been targeted on that unit that it successfully defeats.
6. For each unit in each force, update the status of the unit based on the number of shots that the unit does not defeat. That is, account for the damage a unit sustains by not defeating all the shots.

C. DEFINITIONS

The following definitions are taken from McGunningle (1999). Being a continuation of his work, the definitions and symbology are preserved for the sake of continuity.

Force: A naval surface force, denoted by A or B

Unit: A single warship

Seen target: An enemy unit that is targetable

Shot: A single piece of ordnance targeted at an enemy unit

Good shot: A well-aimed shot that will hit the enemy unit provided the enemy defense does not counter the shot

Shot effectiveness: The probability that a shot is a good shot, denoted by *aS* for force A and *bS* for force B

Firepower: The number of shots a unit can fire in one salvo, denoted by *aF* for force A and *bF* for force B

Striking power: The number of good shots fired by a force in a salvo, denoted by α for force A, and β for force B

Out of action: A unit that has no combat capability remaining, but not necessarily sunk

Unit status: A fraction between 0 and 1 inclusive, describing a unit's capability. 0 describes a unit out of action and 1 a unit with full capability. The unit status is denoted by a for force A and b for force B

Salvo: A salvo is a near instantaneous exchange of shots between force A and force B, denoted by aT for force A and bT for force B

Staying power: The number of hits required to put a unit out of action, denoted by $a1$ for force A and $b1$ for force B

Defensive capability: The weapons, tactics, and employment of defensive measures by a unit to defeat and enemy's good shot, denoted by aC for force A and bC for force B

Defensive effectiveness: The probability that a good shot is defeated by a unit, denoted by aD for force A and bD for force B

Defensive power: The maximum number of shots that a unit will be able to effectively defend against, denoted by $a3$ for force A and $b3$ for force B

Battle: A series of salvo exchanges between two forces, for this work, a battle will be between one and three salvos

Simulation: A battle that has been replicated 1000 times to determine a mean of the fraction of forces killed or surviving as a measure of effectiveness

D. SIMULATION MODELING COMPUTATIONS

As with the definitions, the model computations are taken from McGunnigle (1999). Computations that are changed for exploratory analysis later in the paper will be annotated explicitly.

Indices

i	index of units in force A
j	index of units in force B

Data

aO_i	the initial status of unit i in force A before a salvo is determined
bO_j	the initial status of unit j in force B before a salvo is determined
aC_i	the defensive capability of unit i in force A
bC_j	the defensive capability of unit j in force B
aD_i	the defensive effectiveness of unit i in force A
bD_j	the defensive effectiveness of unit j in force B
aS_i	the shot effectiveness of unit i in force A
bS_j	the shot effectiveness of unit j in force B
aF_i	the firepower of unit i in force A
bF_j	the firepower of unit j in force B
$a1_i$	the staying power of unit i in force A
$b1_j$	the staying power of unit j in force B

Variables

α	the striking power of force A
β	the striking power of force B
$a3_i$	the defensive power of unit i in force A
$b3_j$	the defensive power of unit j in force B

toA_i	the number of good shots targeting unit i in force A
toB_j	the number of good shots targeting unit j in force B
a_i	the status of unit i in force A
b_j	the status of unit j in force B
u	a random variable from a random uniform distribution, U[0,1]

The detailed computations are annotated below. A simplified explanation of the equations is given here, which examines force A shooting at force B for one salvo. Force B shooting at force A is similar. First, the number of shots fired by force A is determined by multiplying the firepower of a unit by its status, for each ship. Because it is nonsensical to fire a fraction of a missile, this number is forced to be an integer though a comparison with a uniform random number. Next, it is assumed that not every missile that is can be being fired will be a good shot. For example, a missile may fail due to a mechanical failure or a poor targeting solution. To compute the number of well-aimed shots by a ship, another random number comparison is made. If the random number is less than the unit's shot effectiveness, then the result is a well-aimed shot, and the force striking power, α, is incremented by one. The total number of well-aimed shots by A is obtained by summing over all its active ships.

Each of these well-aimed shots is randomly assigned to the ships in force B using a uniform distribution. For example, if force B consists of four ships, then the probability that a well-aimed shot targets a ship in force B is 1/4 for each ship in force B. Next, the number of well-aimed shots force B can defeat is calculated by multiplying the defensive capability of a ship in force B by its status, for each ship in force B. As with number of capable shots, this number must also be an integer. It is forced to be such by a comparison with a uniform random number. For each ship, for each well-aimed shot that targets it, another random number comparison is made. If the random number is less than the unit's defensive effectiveness, then the shot is defeated and the defensive power of the ship is incremented by one. This is repeated until all incoming missiles are defeated or the ship exhausts its defensive shots. If a ship runs out of defensive shots, then any remaining shots are assumed to hit the ship.

After the above calculations, the result of the salvo is calculated to update the status of each ship. The new status of the ship is the number of shots that were targeted on a ship, toB_j, minus the number of shots the ship defeated, $b3_j$, divided by the staying power of the ship, $b1_j$. This is subtracted from the initial status of the ship to determine its new status. If this number is greater than, or equal to, one, then the ship is unharmed by the salvo. If it is less than or equal to zero, then the unit has been put out of action by the salvo. If it is between zero and one, the ship is still operational, but functions at a reduced capability for the next salvo.

To compute a second salvo, the parameters are reset to their initial values, with the exception of the unit's status, and the computations are repeated.

The following is the mathematic derivation of how a salvo is computed:

Formulation (from McGunnigle (1999))

1 - Determine the number of good shots fired by a force in a salvo.

1a - Calculate the number of shots each unit in force A is capable of firing based on its status.

For all i = 1….i,

Shots = $aF_i * a_i$

If ($aF_i * a_i$) is not an integer, then

Shots = $aF_i * a_i$ - the decimal portion of $aF_i * a_i$

If u < the decimal portion, then Shots = Shots + 1 (note: u is a uniform [0,1] random number)

1b - Calculate the number of shots each unit in force B is capable of firing based on its status.

For all j = 1…j,

Shots = $bF_j * b_j$

If $(bF_j * b_j)$ is not an integer, then

Shots = $bF_j * a_j$ - the decimal portion of $bF_j * b_j$

If u < the decimal portion, then Shots = Shots + 1

1c - Calculate the number of good shots each unit in force A fires.

For all i = 1….i,

For each unit, the number of shots fired is the result from 1a

For each shot fired, if $u < aS_i$ then the shot is good and $\alpha = \alpha + 1$

1d - Calculate the number of good shots each unit in force B fires.

For all j = 1…j,

For each unit, the number of shots fired is the result from 1b

For each shot fired, if $u < bS_j$ then the shot is good and $\beta = \beta + 1$

2 - Determine the distribution of the force salvo to its opponent's units.

toA_i and toB_j are determined randomly by assigning each of the good shots from each force, α and β, to a unit in the opponent's force. Each unit has the same probability of being targeted by any good shot from its opponent.

3 - Determine how many good shots each unit can defeat.

3a - Calculate the number of good shots each unit in force A is capable of defeating based on its status.

For all i = 1…i,

Good shots = $aC_i * a_i$

If ($aC_i * a_i$) is not an integer, then

Good shots = $aC_i * a_i$ - the decimal portion of $aC_i * a_i$

If u < the decimal portion, then Good shots = Good shots + 1

3b - Calculate the number of good shots each unit in force B is capable of defeating based on its status.

For all j = 1…j,

Good shots = $bC_j * b_j$

If ($bC_j * b_j$) is not an integer, then

Good shots = $bC_j * b_j$ - the decimal portion of $bC_j * b_j$

If u < the decimal portion, then Good shots = Good shots + 1

3c - Calculate the number of good shots each unit in force A defeats.

For all i = 1…i

For each shot the unit is capable of defending against, the result from 3a, if $u < aD_i$, then the shot is good and $a3_i = a3_i + 1$

3d - Calculate the number of good shots each unit in force B defeats.

For all j = 1…j

For each shot the unit is capable of defending against, the result from 3b, if $u < bD_j$, then the shot is good and $b3_j = b3_j + 1$

The result of the salvo exchange affects each unit's status as follows:

$$a_i = a0_i - \frac{toA_i - a3_i}{a1_i} \qquad \text{for all } i = 1\ldots i$$

If $a_i < 0$ then $a_i = 0$

If $a_i > 1$ then $a_i = 1$

$$b_j = b0_j - \frac{toB_j - b3_j}{b1_j} \qquad \text{for all } j = 1\ldots j$$

If $b_j < 0$ then $b_j = 0$

If $b_j > 1$ then $b_j = 1$

An example engagement can be found in McGunnigle (1999), Appendix B.

E. INTRODUCTORY EXPLORATORY ANALYSIS

1. The Basic Scenario

Many battles are simulated with one, two, and three salvo exchanges. Each battle consists of one hundred twenty cases with ship parameters of force size between two and six, defensive capability from one to three, firepower from one to four, and staying power of one and two. Thus, for each battle 120 cases are observed. These are the cases McGunnigle analyzed. For each case, every ship holds identical parameters, representing the homogenous case. The measure of the battle is the fraction of blue forces surviving and the fraction of red forces put out of action. Figure 1 shows the individual outcomes for 1000 replications to obtain statistics on the outputs (from Lucas and McGunnigle (2003)). The final statistics are the means for the fraction of blue forces surviving and the fraction of red forces out of action. Graphs produced show the means of 120 data points for each battle, derived as shown in Figure 1. The high number of replications ensures that the estimated means are very accurate. Specifically, the standard error for an estimate is less than .016.

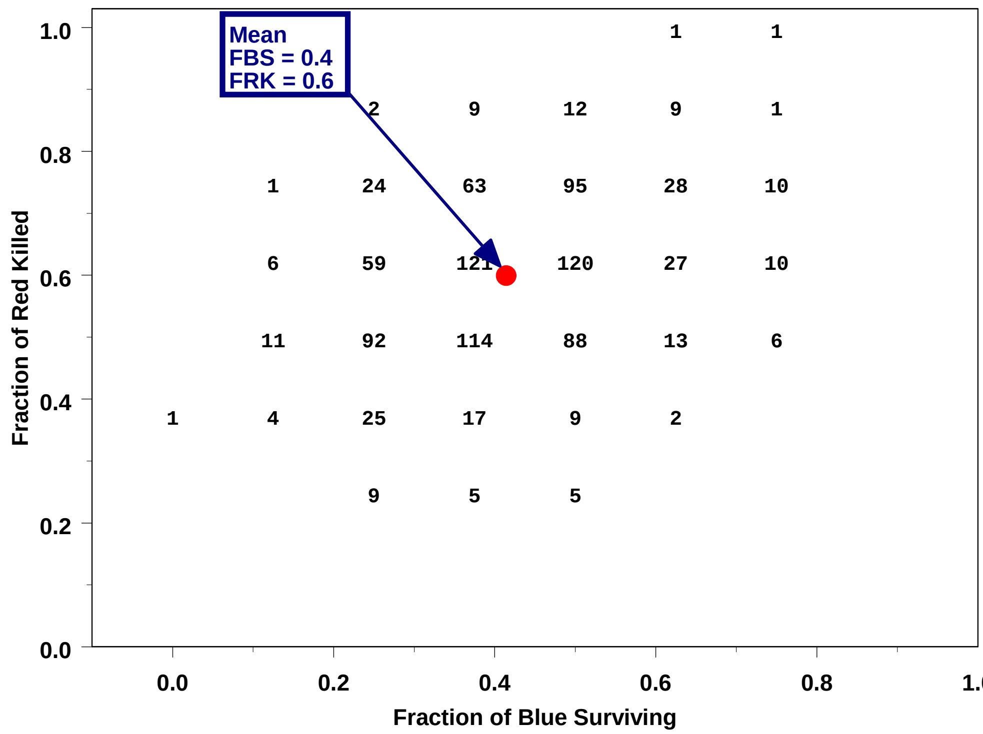

Figure 1. Computing a mean of fraction of blue forces surviving and fraction of red forces killed (From Ref. 3)

2. Basic Scenario Results and Interpretation

As the results of the battle are in terms of fractional values, the scale of both axes is from zero to one. Data points in the upper left portion of the graph represent battles of mutual destruction of the forces. These data points proceed linearly from the upper left corner to the lower right corner representing varying levels of effectiveness of each force, ending at the lower right corner in which neither force is successful in putting opponents out of action. Since the sides are symmetric in these runs, the outcomes must fall on this line. Data points that fall above this line will represent battles in which the blue forces have gained an advantage over the basic scenario; conversely, blue performs worse in data points shown below the line.

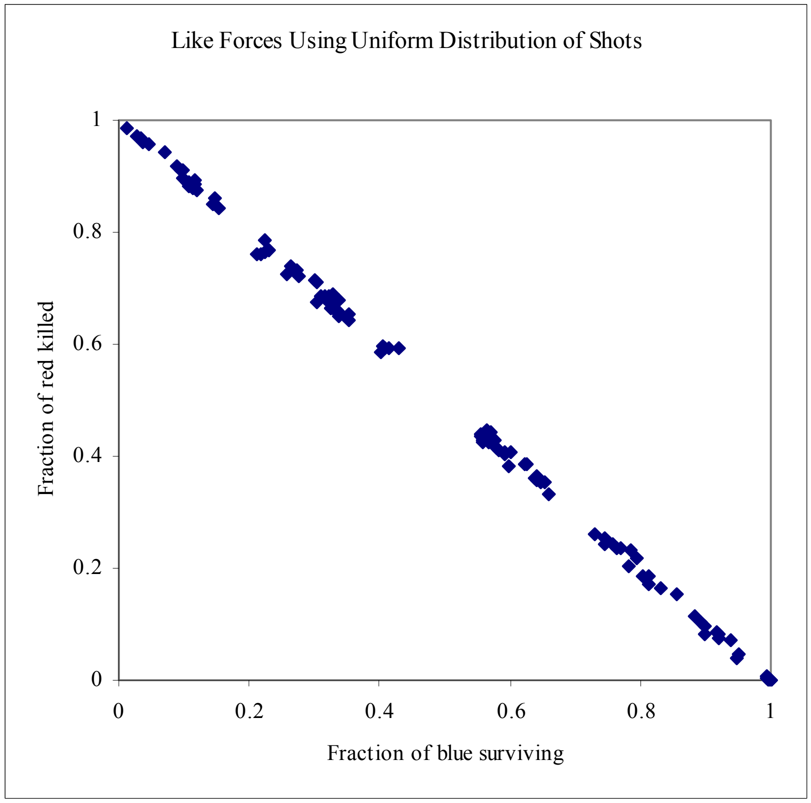

Figure 2. Base scenario showing the means of 120 cases of three salvos for fraction of blue forces surviving to fraction of red forces out of action

If the model is unbiased and blue and red forces each have an equally likely chance of defeating the opponent, since the measure of effectiveness for red is the complement of blue, then the distributions should be nearly identical, only rotated around the midpoint. This is illustrated in the following two distributions, the first being the distribution of the means of the 120 battles of blue forces surviving, the second being the distribution of the means of the 120 battles of red forces out of action. These graphs also illustrate that even though it is possible to achieve the extreme of each side eliminating the other; it is far more likely that some fraction of the forces on both sides will be eliminated.

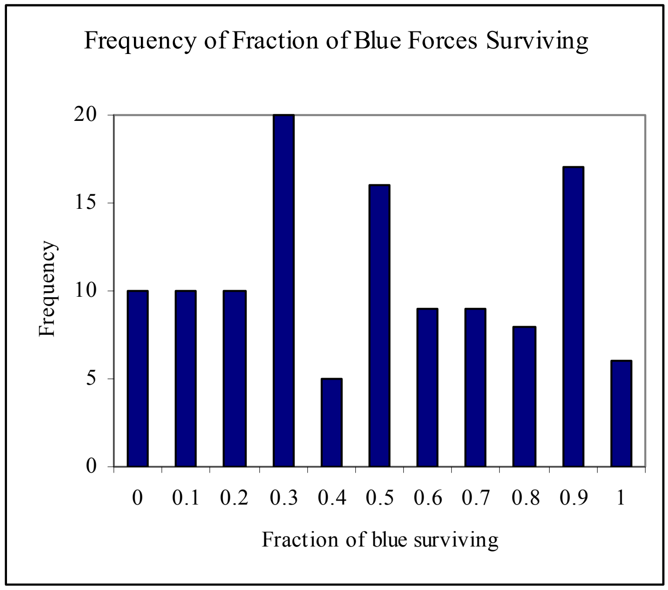

Figure 3. Base scenario distribution of fraction of blue forces surviving in the three salvo battle for 120 cases

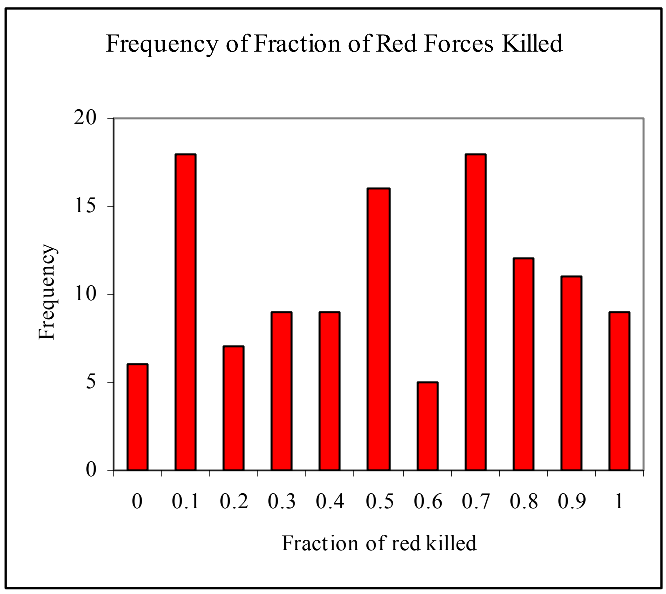

Figure 4. Base scenario distribution of fraction of red forces out of action in the three salvo battle for 120 cases

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III. APPLICATION AND OUTCOMES

A. SINGULAR EFFECTS OF FORCE SIZE ADVANTAGE, INFORMATION ADVANTAGE, FIREPOWER ADVANTAGE, STAYING POWER ADVANTAGE, AND DEFENSIVE POWER ADVANTAGE FOR HOMOGENOUS FORCES

This section examines the effects of singular advantages using McGunnigle's application of the Stochastic Salvo Model. A force size advantage is examined for a one ship advantage. Firepower, defensive, and staying power advantages are incremented by one for the blue forces. An information advantage makes the assumption that the blue force has perfect information on the last salvo's damage executed on red forces, allowing blue to not waste shots on the red units who are already out of action. The other assumptions made are identical to those covered in Chapter II, Section B. Because blue forces are given advantages, blue is expected to perform better, but do specific combinations present better advantages than others over the range of parameters explored? Data points presented in Figures 5 to 9 represent the means of the fraction of blue forces surviving to the means of the fraction of red forces killed for each of 120 cases, each case being a three salvo engagement replicated 1000 times.

1. Effects of Force Size Advantage

For each of the 120 cases, blue has one additional ship in their force. Figure 5 shows that on average the blue forces are much more effective in killing red, shown by the shifting of the data points over the 80% of red forces killed. The cases at the lower right corner, in which blue has high survivability but is unable to defeat red, occur when there are only two or three ships per side, combined with high defensive and staying power and a low offensive power.

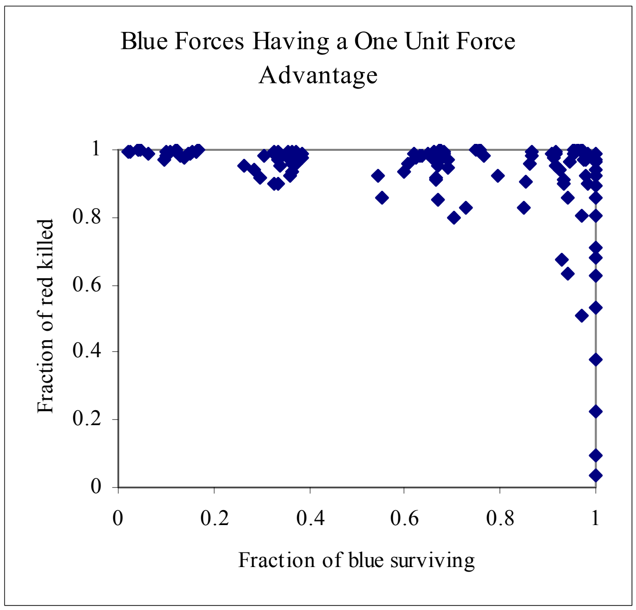

Figure 5. Blue forces having a one unit force advantage for a uniform distribution of shots

2. Effects of Information Advantage

For each of the 120 cases, blue has an information advantage in the ability to conduct perfect battle damage assessment following its shots. In the base case no damage assessment is performed. Figure 6 shows that for the majority of cases the blue forces slightly increase their ability to destroy red. The difference occurs in the middle portion of the data. If the value of firepower is large, but less than the sum of the defensive power and staying power, blue performs better. As firepower and the sum of defensive power and staying power reach near parity, the information yields small to negligible advantages for blue. This illustrates a stamina case, in which for a large number of offensive weapons, the value of the information allows blue to focus its shots to slowly attrite red forces over time, thereby reducing the number of missiles that blue will be forced to counter in turn.

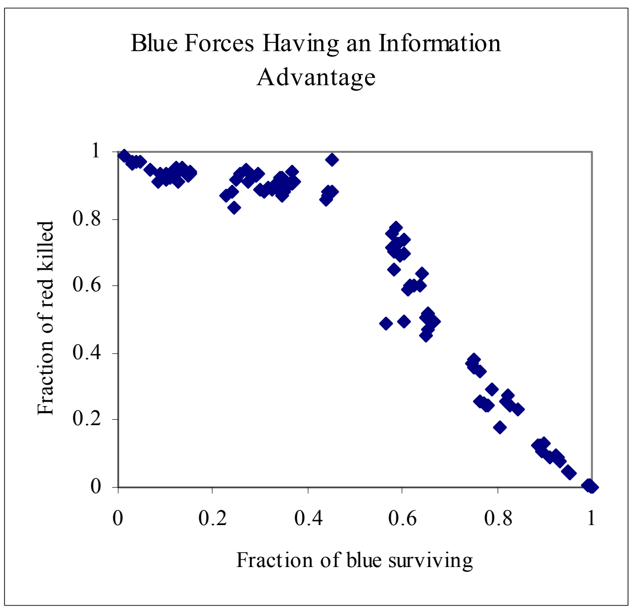

Figure 6. Blue forces having an information advantage for a uniform distribution of shots

3. Effects of Firepower Advantage

For each of the 120 cases, blue has a firepower advantage in that it can fire one additional offensive missile per ship than red. Figure 7 shows the prominent affect of offensive power. For the cases in which forces have the maximum firepower of four, combined with minimum staying power and defensive power values of one, blue destroys red, while suffers heavy losses to its own force. Conversely, the only times when red escapes with an average of less than 20% of its force destroyed is when firepower is minimum, staying power and defensive powers are maximum, and the battle is limited to a two ship versus two ship scenario. For most of the remaining cases, blue forces are successful in eliminating more than 80% of red on average.

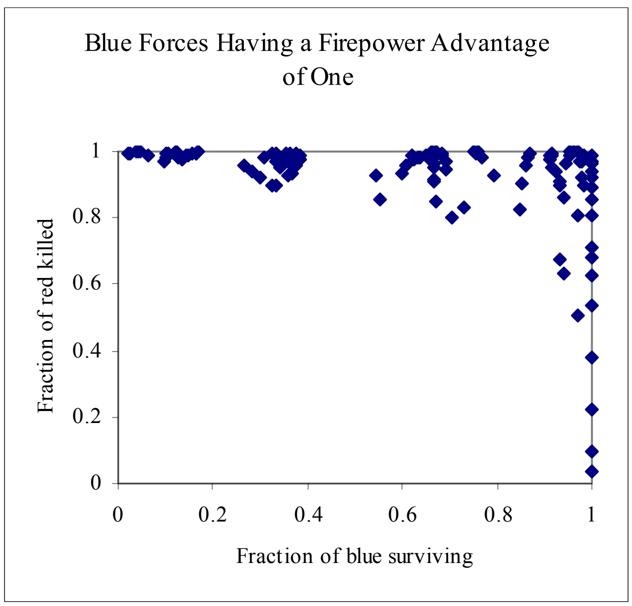

Figure 7. Blue forces having a firepower increased by one for a uniform distribution of shots

4. Effects of Staying Power Advantage

For each of the 120 cases, blue has a staying power advantage of one. Each blue ship can absorb one additional missile before being put out of action. Figure 8 shows the frustration that the red forces have in defeating blue. There is only a single case in which on average more than 80% of blue is killed. This occurs in a two-versus-two scenario with firepower at four and defensive and staying power at one. As the number of ships increases, the affect is less significant, but the imbalance of firepower to defensive power and staying power continue to be the cause of blue realizing higher losses. For the rest of the cases, blue's capacity to withstand more damage allows it to outlast red, thus increasing its survivability and consequently its ability eliminate more of red.

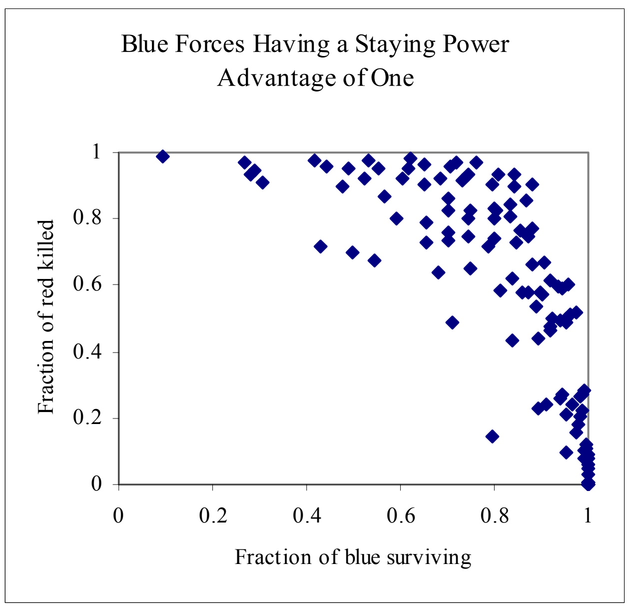

Figure 8. Blue forces having a staying power increased by one for a uniform distribution of shots

5. Effects of Defensive Power Advantage

For each of the 120 cases, blue has a defensive power advantage of one. Each blue ship has one additional opportunity to counter a missile that targets it. Figure 9 shows a similar effect as Figure 8, in fact the same factors that caused blue to be killed in the staying power case are the drivers in the defensive power case. However, increasing defensive power has a more pronounced effect on the ability of blue to kill red. Even though increasing staying power allows the blue forces to remain in the fight longer, blue does so at a reduced capability from the damage it sustains. By increasing defensive power, those good shots are countered before causing damage, thus blue is more effective both offensively and defensively during subsequent salvos.

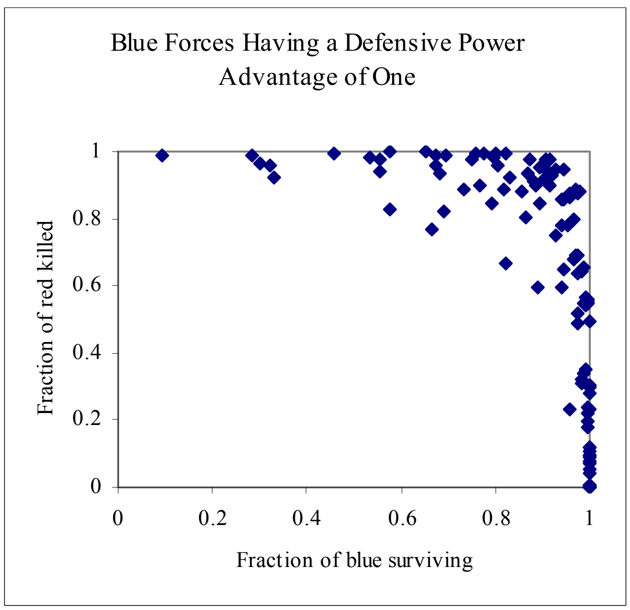

Figure 9. Blue forces having a defensive power increased by one for a uniform distribution of shots

B. PAIRWISE EFFECTS OF FORCE SIZE ADVANTAGE, INFORMATION ADVANTAGE, FIREPOWER ADVANTAGE, STAYING POWER ADVANTAGE, AND DEFENSIVE POWER ADVANTAGE FOR HOMOGENOUS FORCES

This section examines the effects of pairs of advantages using McGunnigle's application of the Stochastic Salvo Model. A force size advantage is examined for a one ship advantage. An information advantage makes the assumption that the blue force has perfect information on the last salvo's damage taken by the red forces, allowing blue to waste no shots on the red forces already out of action. The other assumptions made are identical to those covered in Chapter II, Section B. Firepower, defensive, and staying power advantages are incremented by one for blue forces. Because the blue forces are granted advantages, the question is not whether blue performs better, but rather how much better. Do certain combinations of factors present a better advantage than others? Data points presented in Figures 10 to 19 represent the mean of fraction of blue forces surviving to the fraction of red forces killed for each of 120 cases, each case being a three salvo engagement replicated 1000 times.

As in the singular effects cases, some common trends are observed in the results. In cases in which the number of ships is low, firepower is at the maximum value of four, and defensive and staying powers are at the minimum values of one, blue forces will on average suffer high losses, as the significance of their advantages is less due to the ability of red to eliminate blue units in a single salvo. This is defined as the imbalance case. When the number of ships is low, firepower is at the minimum value of one, and defensive and staying powers are at the maximum values of three and two respectively, blue is unable to execute substantial damage to the red forces, and both sides are frustrated in combat, this is defined as the frustration case. When these parameters are more closely balanced, the blue advantages have a larger effect on the outcome.

1. Effects of Force Size Advantage and Information Advantage

For each of the 120 cases, blue has one additional ship in their force and possess an information advantage with the ability to conduct perfect battle damage assessment following its shots. Both the frustration and the imbalance effects are present. As the parameters become more balanced with respect to the imbalance case, blue is much more effective in eliminating red. As the parameters become more balanced with respect to frustration, blue realizes a higher level of survivability.

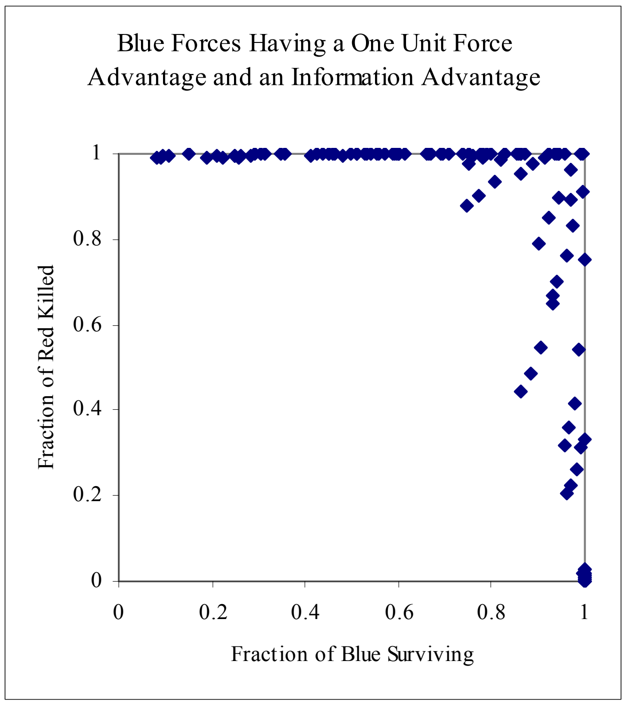

Figure 10. Blue forces having perfect information and a one unit force advantage for a uniform distribution of shots

2. Effects of Force Size Advantage and Firepower Advantage

For each of the 120 cases blue is given an additional ship in their force and each blue ship can fire one more missile (i.e., an increased firepower of one) than its red opponent. The added effect of offensive power is clear in Figure 11. In the majority of cases, blue successfully eliminates all of red; however, it does so while taking significant losses to its own forces in many cases. Even with its significant offensive advantage, blue cannot overcome the imbalance that results from a high firepower with respect to low staying power and defensive power, which are the cases in which blue suffers large losses. However, with this combination, the frustration case is completely avoided, as the offensive advantage allows blue to overwhelm red regardless of the parameters chosen. This is the killer scenario in which a larger force with high offensive power commits to the destruction of its opponent, even at the risk of high losses to its own force.

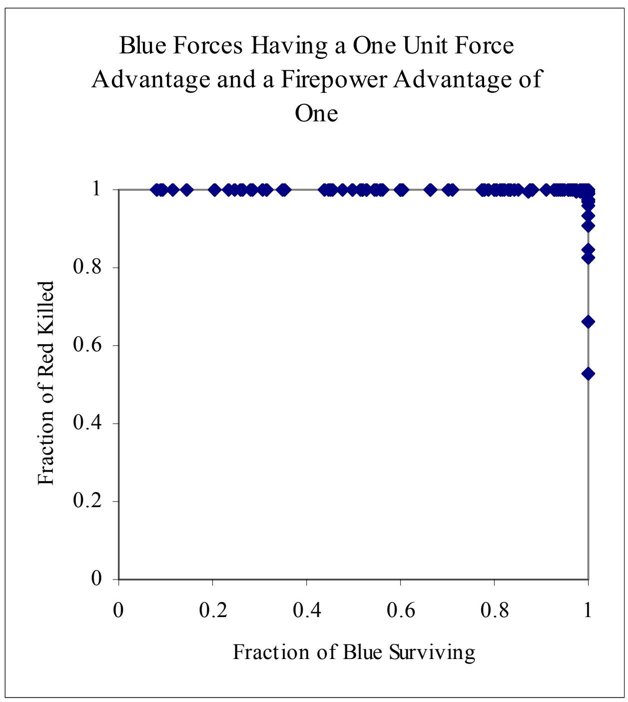

Figure 11. Blue forces having a one unit force advantage and a firepower increased by one for a uniform distribution of shots

3. Effects of Force Size Advantage and Defensive Advantage

For each of the 120 cases, blue has one additional ship in its force and each ship has the opportunity to defend against one additional red force missile per salvo. This contrasts the previous case in that the imbalance case is not present, but the frustration case is pronounced. There are essentially two outcomes, either red is completely destroyed while blue survives with more than half its force, or all of blue survives with some fraction of red being killed. This combination depicts a scenario in which blue can fight with small risk to its own forces, but may be unsuccessful if the primary goal is the destruction of red.

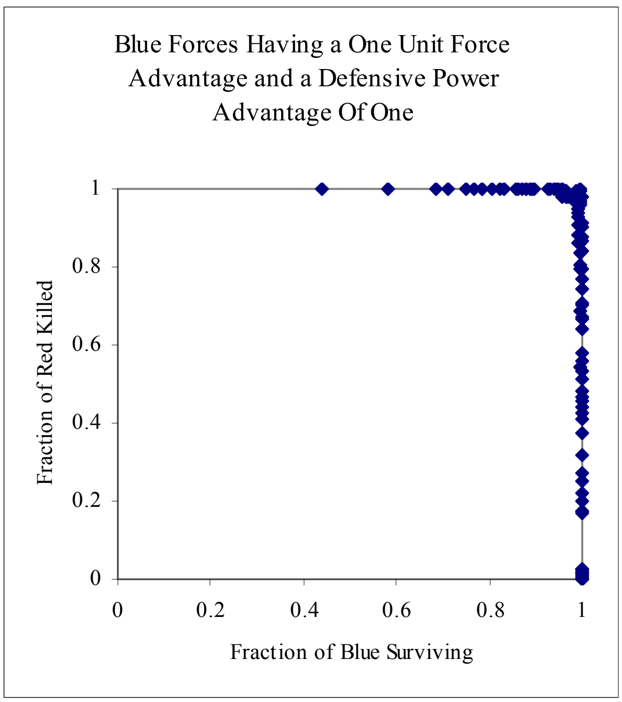

Figure 12. Blue forces having a one unit force advantage and a defensive power increased by one for a uniform distribution of shots

4. Effects of Information Advantage and Firepower Advantage

For each of the 120 cases, blue possesses an information advantage in the ability to conduct battle damage assessment following its shots. Each blue ship can fire one more missile than its red opponent. Figure 13 shows that the imbalance is prevalent in that a large portion of blue can be defeated, but that the frustration cases, although realized, occur to a lesser degree. This would be a viable alternative to the force and firepower combination if the cost to acquire the information would be less than the cost to increase the number of forces, but the end result is similar—destroy red forces, regardless of the loss to blue.

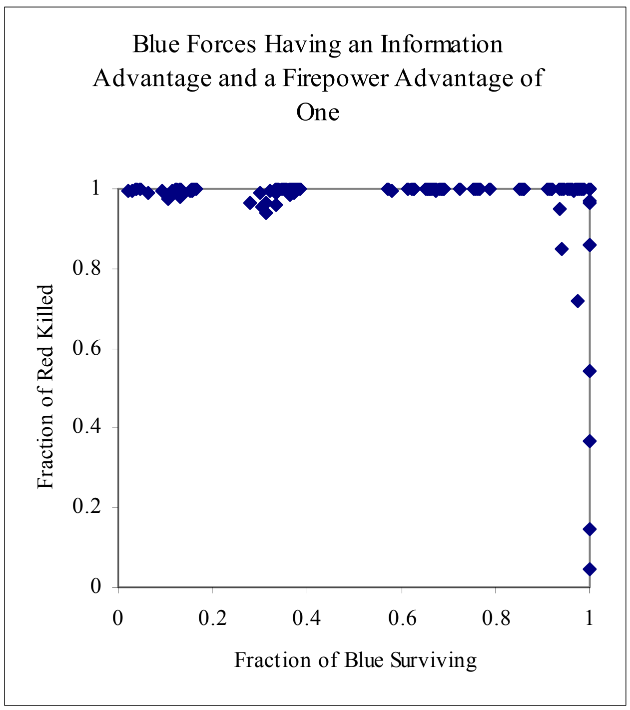

Figure 13. Blue forces having perfect information and a firepower increased by one for a uniform shot distribution

5. Effects of Information Advantage and Defensive Advantage

For each of the 120 cases, blue possesses the information advantage of perfect battle damage assessment following its shots. Moreover, each blue ship has the opportunity to defend against one additional red force missile. Figure 14 shows results similar to the force and defensive power advantage. As with the information and firepower cases, this scenario would be a good selection if the cost for acquiring the information would be less than the cost of acquiring more forces, understanding that in the process blue will realize a slightly higher attrition on average than choosing the force advantage option.

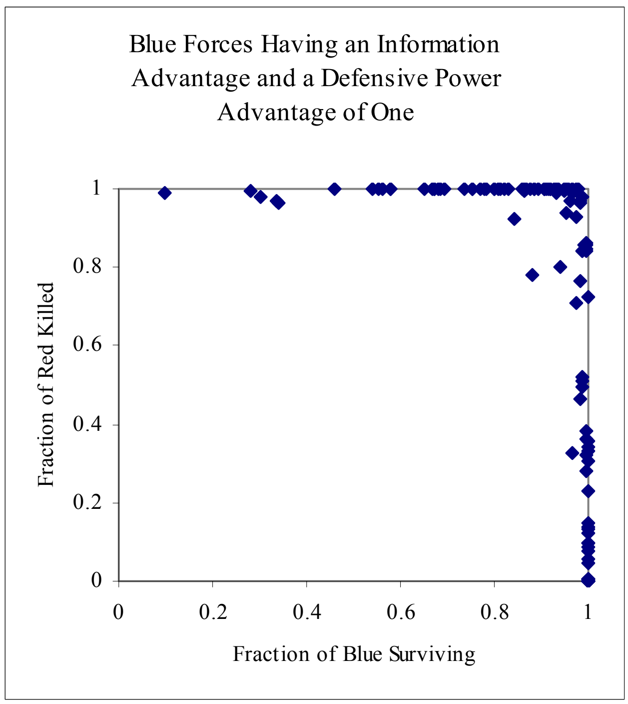

Figure 14. Blue units having perfect information and defensive power increased by one for a uniform shot distribution

6. Effects of Firepower Advantage and Defensive Advantage

For each of the 120 cases, each blue ship is able to fire one more offensive missile and has the opportunity to counter one additional red missile than its red opponent for each salvo. This combination strikes a balance in that the imbalance effect and the frustration effect are both less likely. Additionally, blue has a high level of survivability as well as a strong ability to destroy red.

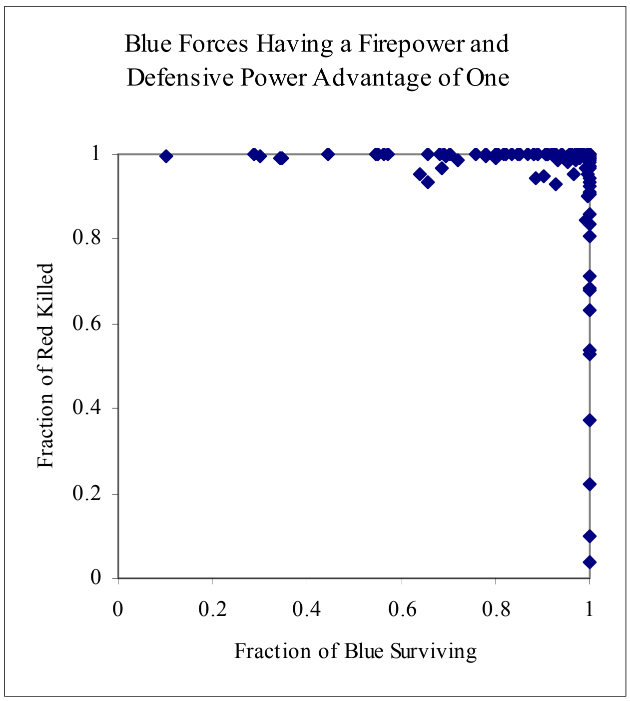

Figure 15. Blue units having a firepower and defensive power increased by one for a uniform shot distribution

7. Effects of Staying Power Advantage and Information Advantage

For each of the 120 cases, blue possesses a staying power advantage in that each blue ship can sustain one additional missile than a red ship before being put out of action. Additionally, blue is given an information advantage; it can conduct perfect battle damage assessment following its shots. Figure 16 shows the frustration effect is prevalent, but the imbalance effect is less significant. However, when compared to the firepower or defensive power cases in which information is paired, the combination of staying power and information is less effective than either of the alternatives. Although blue can outlast red and cause more damage with its advantages when compared to the base scenario, the effects are not as pronounced as the cases where information is paired with firepower or defensive power.

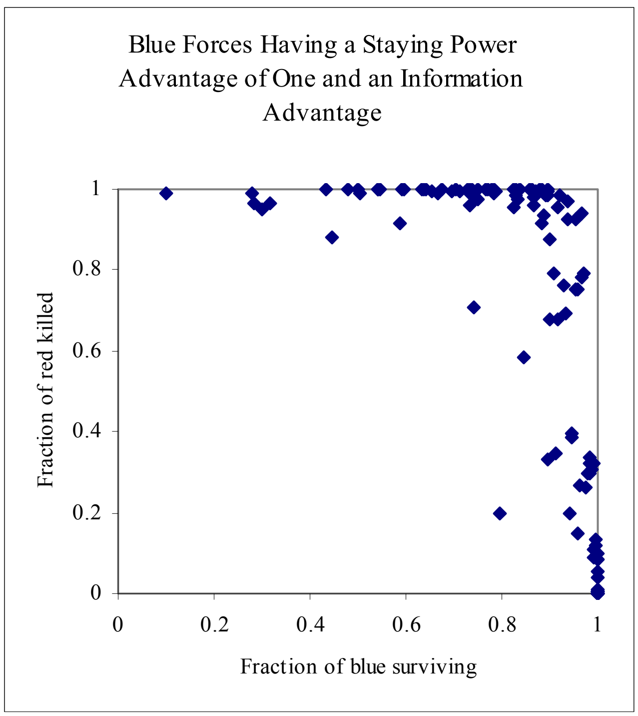

Figure 16. Blue forces having perfect information and a staying power increased by one for a uniform distribution

8. Effects of Staying Power Advantage and Force Size Advantage

For each of the 120 cases, blue possesses a staying power advantage in that each blue ship can sustain one additional missile before being put out of action than red. Additionally, blue is given an additional ship in its force. Figure 17 shows a substantial shift in blue survivability. The imbalance case, in which blue and red are simultaneously destroyed, has shifted such that on average, more than 40% of blue forces will survive while eliminating red. As the parameters become more balanced, the fraction of blue forces surviving increases. However, the frustration effect is pronounced, and although surviving, blue is not as successful in eliminating red.

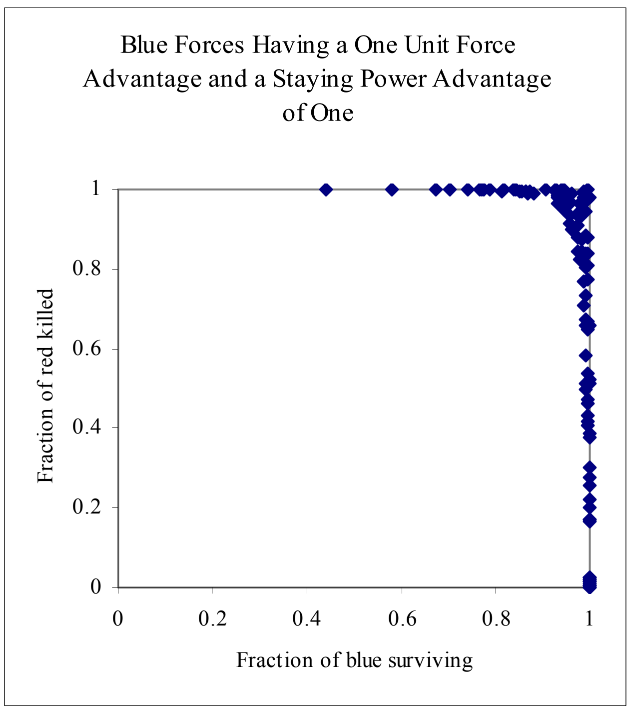

Figure 17. Blue forces having a one unit force advantage and a staying power increased by one for a uniform distribution of shots

9. Effects of Staying Power Advantage and Defensive Power Advantage

For each of the 120 cases, blue possesses a staying power advantage in that each blue ship can sustain one additional missile than red before being put out of action. Additionally, blue is given a defensive power advantage in that each blue ship possesses one additional defensive opportunity to counter a missile that targets it. Figure 18 shows that if both the advantages chosen are designed to increase blue's survivability, the combined effects greatly increase blue's chance of escaping with relatively small losses. A small benefit in the ability to kill red is observed, as blue can outlast red over the three salvos. If it is expected that the battle will last over a period of many salvos, instead of the smaller choice of three, then the stamina of blue allows it to destroy more red units. However, if the battle is expected to be short and decisive, this combination will allow blue to escape relatively unharmed, but this is also true for red.

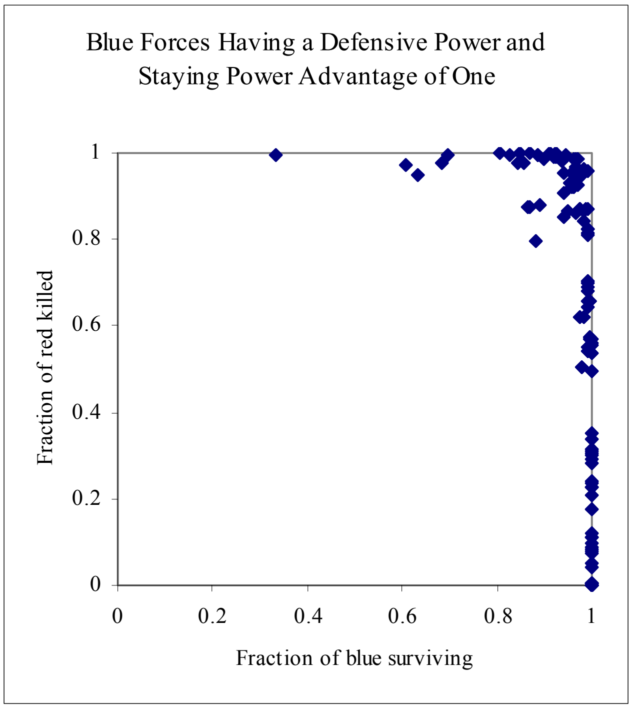

Figure 18. Blue forces having a defensive power and staying power increased by one for a uniform distribution of shots

10. Effects of Staying Power Advantage and Firepower Advantage

For each of the 120 cases, blue possesses a staying power advantage in that each blue ship can sustain one additional missile before being put out of action than red. Additionally, each blue ship is given a firepower advantage in that each blue ship can fire one more missile than red. Figure 19 shows an effect similar to the defensive power and firepower case. The difference is that blue is more likely to counter the missile in the defensive case, but in the staying power case, blue is better able to survive the hit from the missile. Because a hit reduces the effectiveness of a unit, blue is less successful both in preserving its force and in eliminating red. However, the combination of firepower and staying power advantages realize large gains to both survivability and lethality to red. If the cost of enhancing staying power by adding extra armor, adding bulkheads, or other improvements to increase staying power is less than the cost of adding defensive systems, then this case may be preferred to the firepower and defensive power selection.

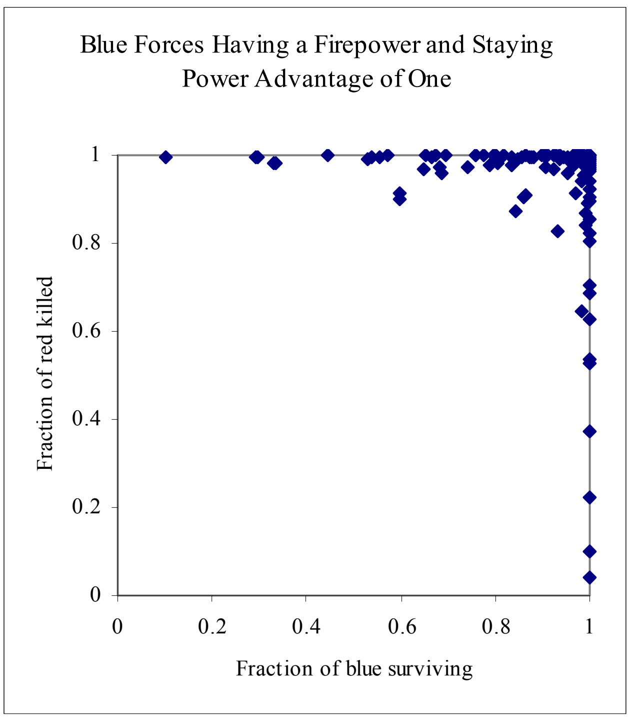

Figure 19. Blue forces having firepower and staying power increased by one for a uniform distribution of shots

C. COMPARING EFFECTS USING THE FRACTIONAL EXCHANGE RATIO

Making a decision of parameters based on a fraction of red forces killed will yield a force focused on overpowering an enemy. But, as shown in Figure 11, this may not be the best policy, as the survivability for blue can be fairly low. Choosing to compare the models with a mean fractional exchange ratio (that is the mean of the fraction of blue forces killed over the 120 cases divided by the mean of the fraction of red forces killed over the 120 cases) allows the decision maker to compare the tradeoff value for a blue kill to a red kill. A fractional exchange ratio of one is analogous to neither side having an advantage. A fractional exchange ratio closer to zero indicates a larger blue advantage. Since the comparisons were designed with a blue advantage, the fractional exchange ratio is expected to be less than one for all cases. Figure 20 shows the fractional exchange ratios for all cases examined. The axis is scaled from zero to one. The pairwise effect cases are identified above the axis, and the singular effect cases are identified below the axis. For example, the fractional exchange ratio for the information and firepower advantage case is shown at .40. This means that although blue realizes an advantage in combat with information and firepower increases, the effect is the least powerful of all the pairwise effects cases. In this case, for every five red units killed, blue can expect to suffer two losses to its own forces.

If making the decision is based purely on the fractional exchange ratio, the decision is to opt for a larger force with a better defensive power. This shows the importance for decision makers to carefully determine the goals for their force. By choosing the force and defensive option, blue can slowly attrite red without suffering large losses. However, what if the situation dictates the rapid destruction of the opponent? For such a match blue forces would likely survive relatively unscathed, but potentially red also escapes with a large portion of theirs, and the mission is failed.

Choosing a combination that strikes a balance between blue surviving and red killed, such as the firepower and staying power combination, will produce a force capable of performing well under most circumstances. But what if the red's only desire is to gain a few casualties to enhance its nation's prestige? Then the decision may again change.

This highlights the role of the contribution of modeling and simulation. Provide the decision makers with the best information that can be provided, and allow them to more knowledgably weigh the risks.

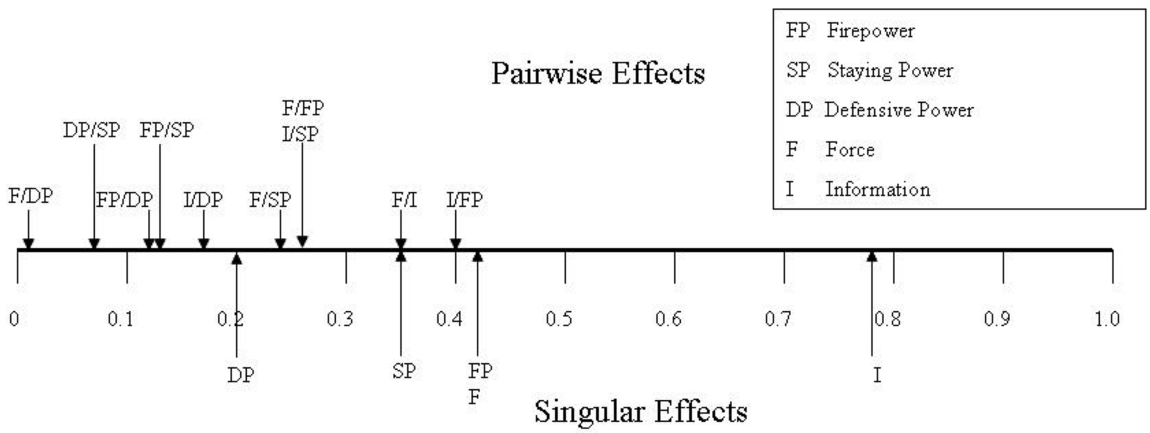

Figure 20. Fractional exchange ratios computed as the mean of the fraction of blue forces killed to the mean of the fraction of red forces killed over 120 cases for singular and pairwise interactions

D. POLYA DISTRIBUTION OF GOOD SHOTS

All the variants of Hughes' model assume a uniformly distributed shot distribution to the opponent's forces. There is evidence that shooters do not behave in this manner. A unit that has a successful engagement is more likely to have a future successful engagement (Bolmarcich, 2000). It is possible that targeting of forces is assigned similarly. Consider a case in which blue knows that a command-and-control ship is in the red force. It is likely this would be a primary target and shots would be allocated to put that unit out of action first. There is also a possibility that geometry dictates one ship as a priority, the closest ship may be the larger threat. Or that shipping exists between the forces providing a better targeting solution for one red unit over another. The Polya distribution captures this effect. This section tests the uniform distribution assumption to examine if a non-uniform distribution produces similar results.

1. The Polya Distribution

The Polya distribution is best explained in the classic urn description. For this explanation, a ship will be analogous to an urn and a well-aimed missile targeting that ship is analogous to a ball. Assume the weight of an urn is one, as is the weight of a single ball. The probability of a ball being distributed to an urn is determined by the sum of the weight of the urns and the sum of any balls that have been previously assigned to urns. Consider the case below in which there are four urns and a total of six balls to distribute among them.

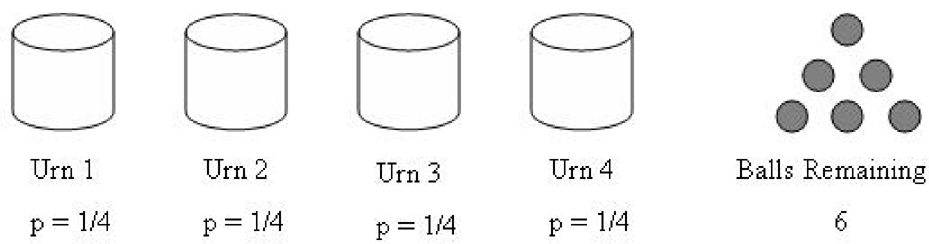

Figure 21. Initial Polya distribution for four urns and six balls

Because there are no balls in any urn, and the sum of the weight of urns is four, the probability, p, of a ball being deposited in any urn is equal to 1/4, i.e., all urns are equally likely to receive the ball. Suppose that the first ball is deposited in urn number two, as shown below.

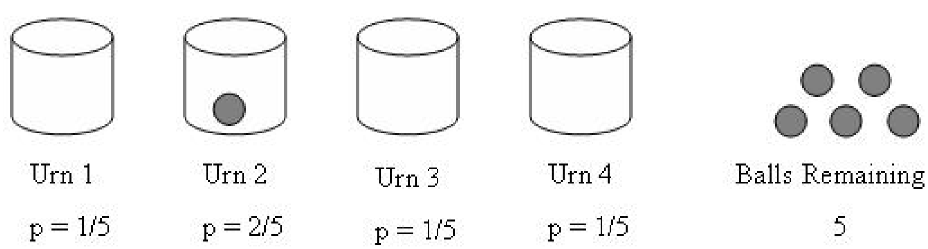

Figure 22. Polya distribution for six balls and four urns after one ball has been distributed

Now the sum of the urn and balls is five. For urns one, three and four, that contain no balls, their probabilities of receiving the next ball are their weights over the total weight, or p = 1/5. Because urn two contains its weight and the weight of its ball, its probability of receiving the next ball is 2/5. This process continues for the second ball.

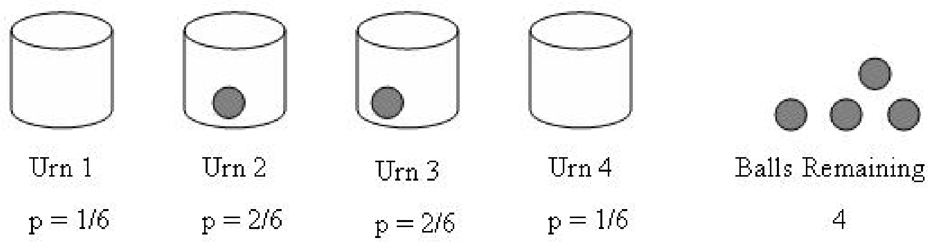

Figure 23. Polya distribution for six balls and four urns after two balls have been distributed

In this case, the second ball is deposited in urn three. With four urns and two balls, the total weight is now six. With urns one and four containing no balls, the probability of their obtaining the next ball is only 1/6, while the probabilities of urns two and three are 2/6. After just two balls have been assigned, the probability of urn one or urn four receiving the next ball is rapidly decreasing. This disparity is exaggerated in the event that urn two had received the second ball as well. If this had been the case, urn two would have had a probability of receiving the next ball of 3/6, while urns one, three, and four would have remained at 1/6. Let's continue with the example, but jump to the case in which there is only one ball left to assign (see Figure 24).

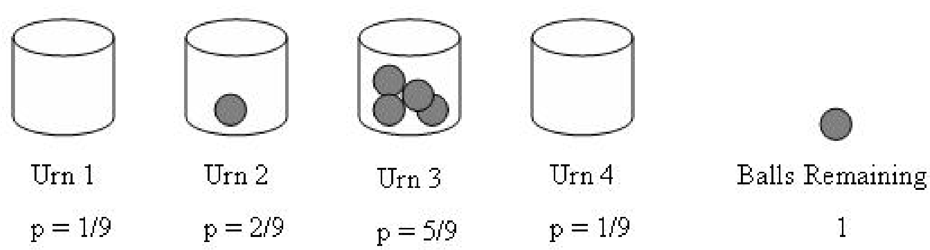

Figure 24. Polya distribution for six balls and four urns after five balls have been distributed

This represents a common occurrence in a Polya distribution. With the third and fourth balls assigned to urn three due to its increased probability of receiving a ball, urn three's probability increases sufficiently so that a ball being assigned to urn one, two, or four is unlikely. One would expect that the final ball would most likely (with probability of 5/9) be distributed to urn three.

2. Implementing the Polya Distribution in the Hughes Salvo Model

The basic structure of the simulation remains the same. The number of shots each ship is capable of shooting is determined. The number of well-aimed shots fired by each ship is computed and summed over the force to determine the force striking power. The change is in the assignment of shots to the opponent force. Following the procedure explained in the previous section, the first shot is assigned with a uniform distribution. However, for the following shots, the probabilities of the targeted force ships are adjusted based on the results of the previous shot. This is continued for all shots in the salvo. Next, as before, the number of shots each ship is capable of defeating is calculated and the number of shots each ship actually defeats is calculated. Finally, the status of each ship is updated. For the second salvo, the procedure is repeated, with the exception that the first shot of the salvo is not uniformly distributed, but is based on the probabilities determined by the last shot of the previous salvo.

3. Effects of a Polya Distribution of Shots with no Information Advantage

A battle is conducted with three salvos. The battle consists of 120 cases with ship parameters of force size between two and six, defensive capability from one to three, firepower from one to four, staying power of one and two, and a Polya distribution for shot assignment. For each case, every ship holds identical parameters, representing the homogenous case. Each battle is replicated 1000 times to obtain a mean for the fraction of blue forces surviving and a mean for the fraction of red forces put out of action for all 120 cases. Figure 25 shows the 120 data points for each battle.

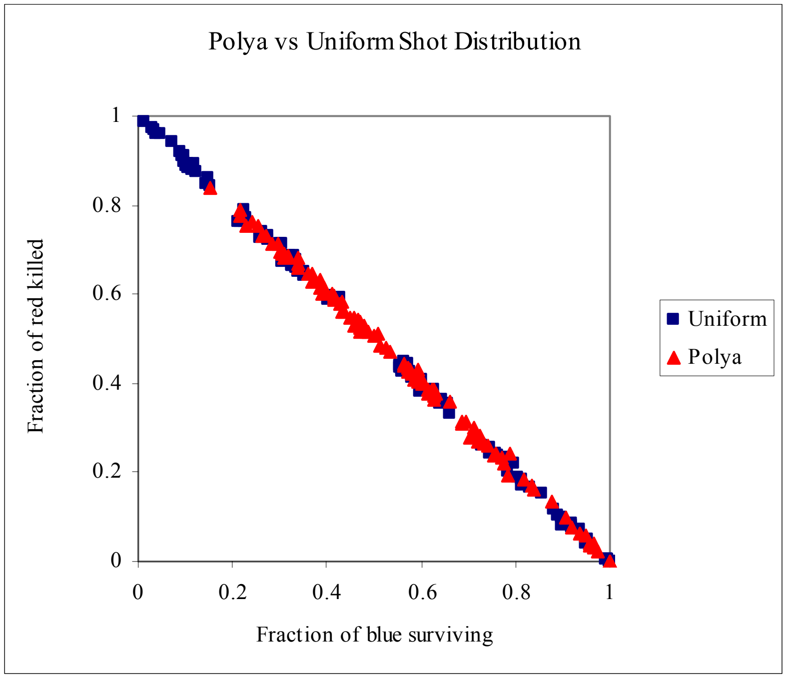

Figure 25. Polya shot distribution for 120 cases compared to uniform distribution of shots for 120 cases for fraction of blue forces surviving and fraction of red forces killed (best viewed in color)

While the uniform shot distribution captures all possibilities from all forces killed to no forces killed, the inherent nature of the Polya distribution modifies these results. Due to the unlikely occurrence that every ship in a force is targeted in a salvo sufficient to eliminate the entire force, on average the highest expected loss is limited to 80% of the force. Although there are occasions in which an entire force can be eliminated, when replicated 1000 times, these occasions are so infrequent that they can not increase the maximum average fraction of forces killed above .8.

Examining the histograms for the frequency of fraction of red forces killed reveals different behaviors in the expected fraction of forces killed. Figure 26 shows that for a uniform distribution the cases are spread rather evenly regardless of the fraction of forces killed. However, looking at Figure 27, there is a strong skew in the distribution where the most common occurrences result in between 30% to 70% percent of the force being eliminated.

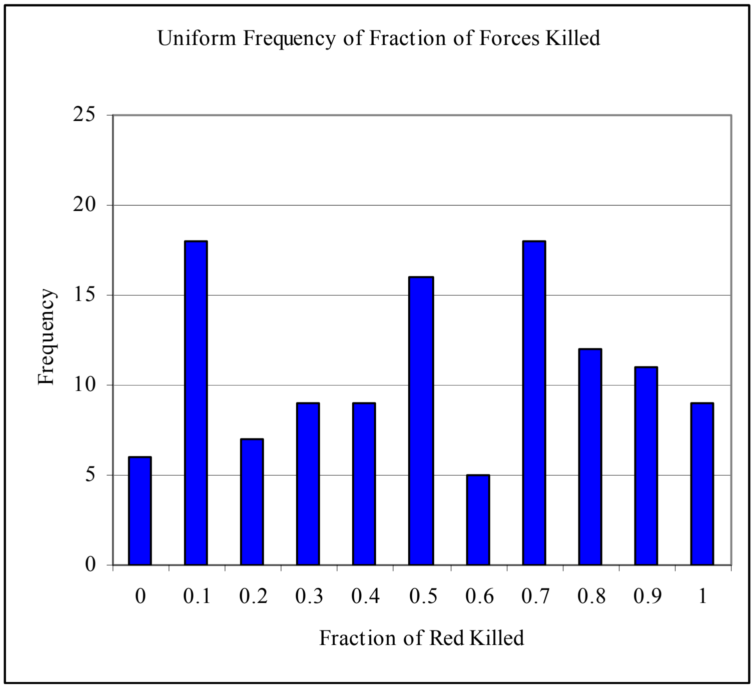

Figure 26. Histogram showing the frequency of the fraction of red forces killed using a uniform distribution of shots

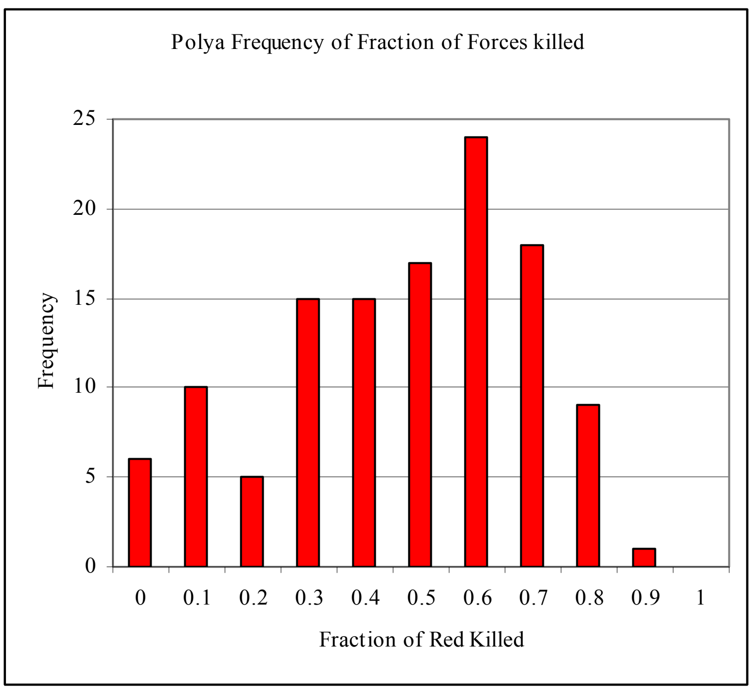

Figure 27. Histogram showing the frequency of the fraction of red forces killed using a Polya distribution of shots

4. Effects of a Polya Distribution of Shots with Blue Force Information Advantage

An information advantage is defined as to a force's ability to perfectly determine battle damage assessment from their shots. As shown in the illustration of the Polya distribution, once a ship has had shots assigned, it begins to dominate the number of shots that are assigned to it in the future. This leads to an exaggeration of the overkill effect in which a force expends more shots on a target than is required to eliminate it. If a force could correct this by properly assessing the damage exerted to the opponent force, it leads to a more optimum allocation of shots, expending only the number of shots required to put the target out of action. There is no requirement for a force to know the defensive and staying power of the opponent force, only that it can properly assess the battle damage. Information in this scenario is assumed to be perfect. That is, the shooter knows exactly when a unit it has targeted is out of action from the last salvo. The information received is instantaneous and not confused with another unit. Figure 28 shows the result of the 120 cases with blue possessing the information advantage allocating shots via a Polya distribution compared to the Polya shot distribution in which neither force has an information advantage.

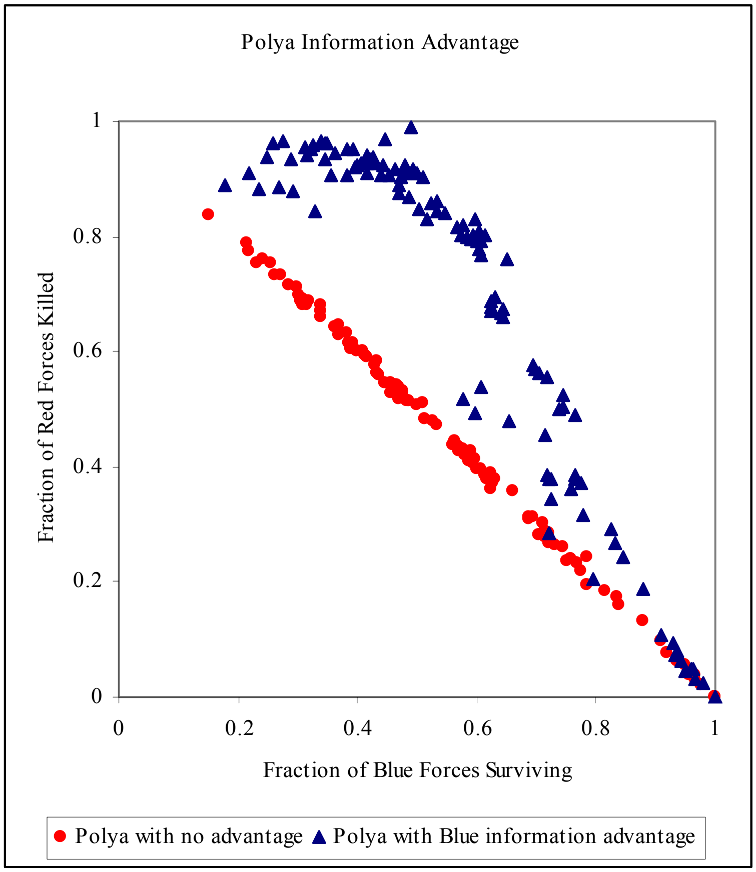

Figure 28. Polya shot distribution comparing 120 cases with blue possessing an information advantage to 120 cases where neither has an advantage, displaying fraction of red forces killed to fraction of blue forces surviving for each case

Clearly, if such information could be achieved, blue gains a significant advantage over the red forces. The clustering of data in the lower right corner illustrates that when no forces are killed, blue can do no worse than the case of no information. However, as indicated by Figure 27, most of the cases occur when the fraction of red forces killed is between the .3 to .7 ranges, so this is not a common outcome. In this range of data, with information, blue possesses a large advantage. Additionally, with the increase in information, blue can break the barrier of on average killing greater than 80% of the red force. If one believes the Polya shot assumption, pursuit of such an information advantage is a worthy effort. Possible opportunities to exploit this include organic unmanned aerial vehicle (UAV) assets focused specifically on providing real-time battle damage assessment, or exploring tactics to deny an enemy this same advantage. Perhaps this could be something as simple as a smoke cloud.

5. Effects of a Polya Distribution of Shots with Both Blue and Red Forces Having Perfect Information

The importance of the information has been clearly shown, but are similar results expected if both units are successful in exploiting this perfect information?

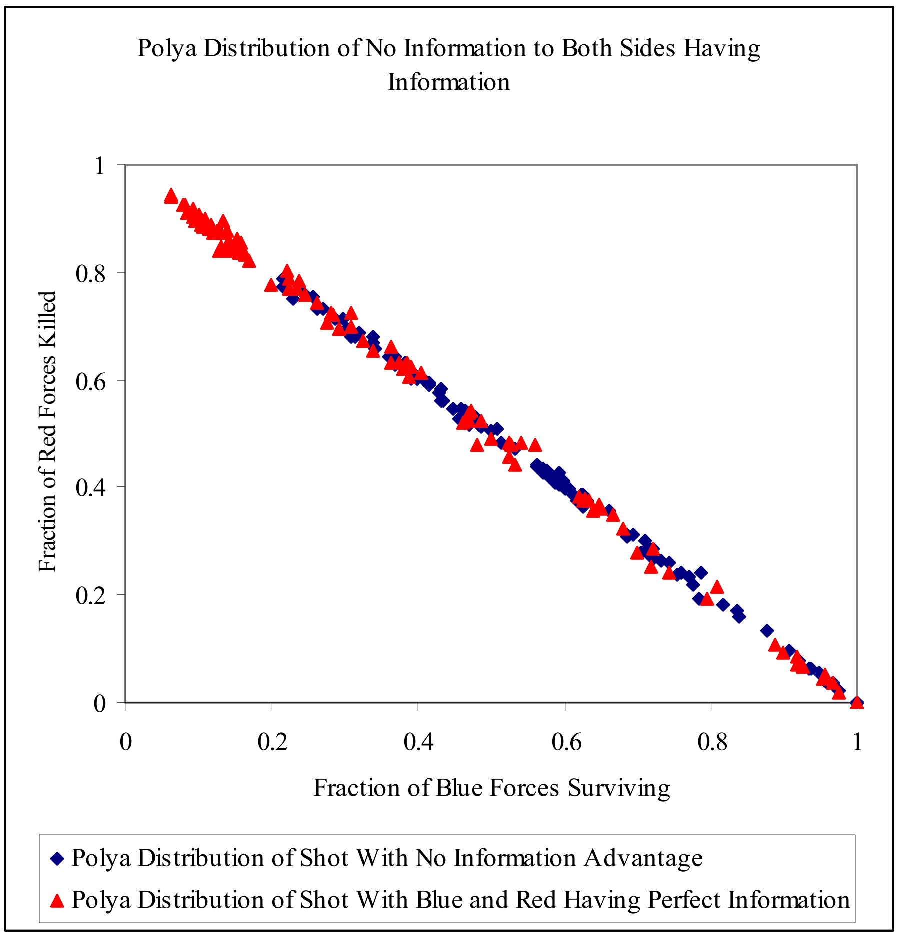

Figure 29. Polya shot distribution comparing 120 cases with neither side possessing an information advantage to 120 cases where both sides have perfect information, displaying fraction of red forces killed to fraction of blue forces surviving for each case

Figure 29 illustrates that with both sides having perfect information, neither has an advantage to assist in defeating the opponent. However, a dangerous situation exists in that combat is much more deadly in this environment.

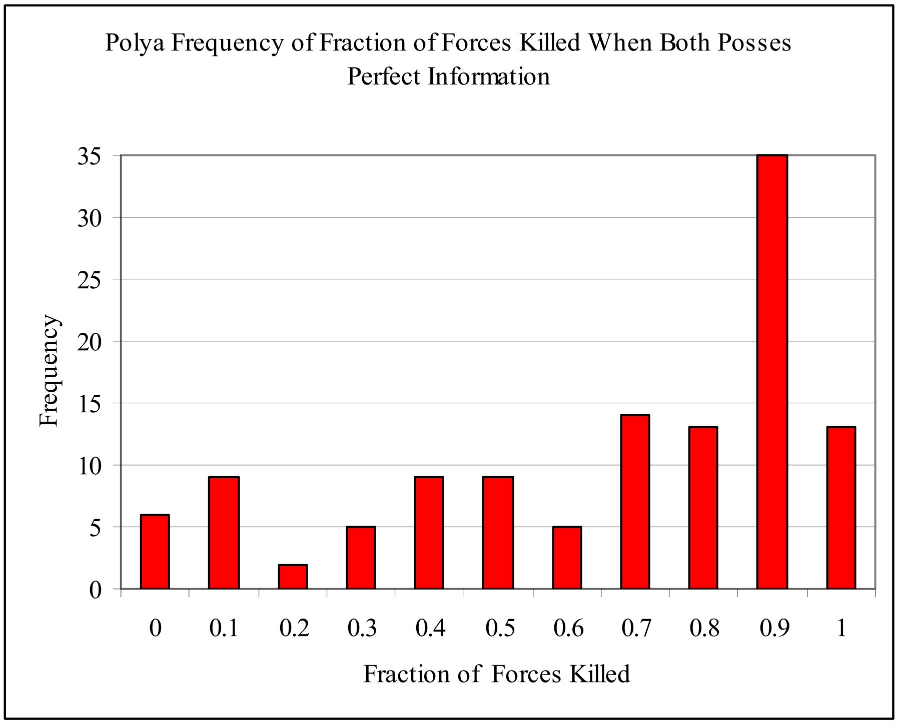

Figure 30. Histogram showing the fraction of red forces killed using a Polya distribution of shot with red and blue forces possessing perfect information

Comparing Figure 30 to Figure 27, the shift to the right is clear. Although the figures represent a fraction of red forces killed, because the forces are identical, the distribution of the fraction of blue killed is similar. This illustrates that even though the benefits of obtaining an information advantage is significant, it is at least as important to simultaneously deny the opponent this same advantage. A force can win with the information advantage, yet if they fail to deny the opponent access to the information on their own force, the destruction of both forces is the most common occurrence.

IV. SUMMARY AND CONCLUSIONS

A. SUMMARY OF RESULTS

1. Pairwise Interactions of Parameters Assigning Shots with a Uniform Distribution

An engagement is simulated using Hughes' Salvo Model to determine the effects of giving the blue force a one unit increase in two different parameters. The ship parameters explored are the number of offensive shots it can fire, the number of defensive opportunities it has to counter the missiles that have targeted it, and the number of missiles it can sustain before being put out of action. The force parameters explored are the number of units in a force and the ability of a force to conduct perfect battle damage assessment on the opponent. For each pair of parameters chosen, the simulation is run for 1000 repetitions to determine a mean fraction of blue forces surviving and a mean fraction of red forces killed for a three salvo engagement. Table 2 shows the results of these ten cases, and the five individual cases.

Blue Advantages	Mean Fraction of Blue Surviving	Mean Fraction of Red Killed
Force	0.62	0.91
Information	0.54	0.59
Firepower	0.62	0.91
Staying Power	0.81	0.54
Defensive Power	0.88	0.59
Firepower Defensive Power	0.89	0.94
Force Defensive Power	0.95	0.73
Force Firepower	0.74	0.99
Force Information	0.72	0.79
Information Defensive Power	0.88	0.7
Information Firepower	0.62	0.97
Staying Power Information	0.83	0.67
Staying Power Force	0.95	0.72
Staying Power Defensive Power	0.95	0.61
Staying Power Firepower	0.88	0.93

Table 2. Summary of means of fraction of blue forces surviving and means of fraction of red forces killed for 120 cases showing the singular and pairwise comparisons using a uniform distribution of shots

Table 2 shows that there are combinations in which blue effectively destroys the red force that are better than others. However, there are also cases in which red defeats a large portion of the blue forces in its defeat. An alternative comparison is the fraction of blue force that is killed to the fraction of red forces that is killed, commonly called the fractional exchange ratio. A fractional exchange ratio of one indicates neither unit has an advantage in the engagement. A fractional exchange ratio that approaches zero indicates a larger advantage for blue. Table 3 shows the results of the fractional exchange ratio over the ten cases explored.

Blue Advantages	Fractional Exchange Ratio
Force	0.42
Information	0.78
Firepower	0.42
Staying Power	0.35
Defensive Power	0.2
Firepower Defensive Power	0.12
Force Defensive Power	0.06
Force Firepower	0.26
Force Information	0.35
Information Defensive Power	0.17
Information Firepower	0.4
Staying Power Information	0.26
Staying Power Force	0.24
Staying Power Defensive Power	0.08
Staying Power Firepower	0.13

Table 3. Summary of fractional exchange ratios from the singular and pairwise effects using a uniform distribution of shots

Table 3 shows that the best scenario occurs when blue concentrates on defensive power and an increase in force size. This is because while a potentially large portion of the red forces are destroyed, blue remains undamaged in most cases. As the battle progresses, blue is able to attrite, but red is largely ineffective at penetrating blue's defensive capability. It is important not to over generalize the significance of these results, as these represent the effect of examining only a one unit increase to any of the parameters. Without running sensitivity analyses, to claim that a single combination of parameters generates the most significant gain in blue performance is dangerous. However, focusing investment on parameters that enhance survivability offers promise.

What can be declared is that over the range of data observed, if the primary goal weighs the value of a blue surviving higher than that of a red kill, then the force and defensive power combination is the best alternative. This statement contains two qualifiers. First, one must accept the range of data as acceptable and within the range that will likely be encountered in a real world scenario. Second, the measure of performance is clearly laid out. If this measure changes to, say, defeat all red units and accept a predetermined acceptable level of loss for blue forces, then a different selection of parameters is recommended. This is why decision making is difficult, and the role of modeling is to provide the decision makers with the most complete, accurate, and concise data possible, so that they can make the most informed, and hopefully best, decision possible. As tables 2 and 3 show, even small differences in capability can result in big differences in outcome.

2. Using a Polya Distribution for Assignment of Shots

An assumption made in Hughes' Salvo Model is that shots are uniformly distributed among the opponent force. That is, each ship is equally likely of being targeted by any missile. An engagement is simulated using Hughes' Salvo Model to determine the effects of assigning shots to the opponent force using a Polya distribution. The Polya distribution weights ships that have been targeted previously as being more likely to continue to be targeted. This mirrors a tactic of concentrating fire on one enemy to defeat it before selecting the next target. There is still a chance that a different opponent ship will be targeted, but as more shots target the original target, the probabilities of such a case decrease. 120 cases are simulated between like forces varying the parameters of each force by case. The ship parameters explored are the number of offensive shots it can fire, the number of defensive opportunities it has to counter the missiles that have targeted it, and the number of hits it can withstand before being put out of action. The force parameters explored are the number of units in a force, and the ability of a force to perfectly determine the battle damage assessment of the opponent to prevent wasting shots. The simulation is run for 1000 repetitions to determine a mean fraction of blue forces surviving and a mean fraction of red forces killed for a three salvo engagement.

The assumption of a uniform shot distribution allows for the full spectrum of results, from both forces being destroyed to neither force able to exact damage on the opponent, with any case being equally likely. With a Polya distribution of shots, this changes because no more than eighty percent of any force can be defeated, on average. The most frequent occurrence shows that between 30 to 70% of the red force is expected to be destroyed. This is due to the nature of the Polya distribution in that a ship that has been destroyed will continue to draw fire until the end of the engagement, exacerbating the overkill effect that can occur in Hughes' Salvo Model. To correct for this, blue is given an information advantage.

Blue forces are given the benefit of being able to perfectly determine a battle damage assessment of the opponent forces after each blue missile is fired. This corrects the problem with the Polya distribution in that a unit will only be targeted until it is put out of action. After this, salvos will be allocated to the remaining ships. Thus, missiles are not wasted on units that are already eliminated.

The results (see Figure 28) show that when blue is given an information advantage it can do no worse and in most cases performs better than it did in the case with no information. Additionally, blue can defeat more than 80% of the red forces in many scenarios, demonstrating that for identical forces the information advantage alone is sufficient to tip the scale of the battle in favor of blue.

The previous example showed the importance of being able to diagnose the effects of the previous shot, denying the enemy the ability to assess his shots is similarly important. Figure 29 illustrates that although neither holds an advantage in combat, the results are more deadly for both sides. In over sixty percent of the cases, both forces will suffer losses of more than seventy percent.

Therefore, for a targeting tactic in which targets are prioritized, being able to ascertain the results of prior engagements is significant to defeat more of the opponent force. However, if it is believed that the opponent is also following this same strategy, effort must be made to deny this information, or more blue force casualties will be incurred in order to destroy the enemy.

B. RECOMMENDATIONS FOR FOLLOW ON RESEARCH

1. Increasing Range of Parameters

Exploring a single unit increase over the parameters reveals how the variables interact. However, there is a point at which increasing the parameter further is not reasonable. For example, staying power, which was shown as a vital parameter to keep balanced, is probably infeasible to be increased to a value of (say) ten. A ship able to withstand ten missile hits and not be put out of action would probably be cost prohibitive to build. The next question then becomes, how high can offensive power or force be pushed so that staying power remains relatively balance?

Additionally, what if the single parameters are increased to more than one unit of advantage, how does it affect the results? Past a certain point it will be difficult to make comparisons as it is likely that all of blue will survive and all of red will be killed. But could it be so significant that the same results would be realized over a single salvo? If so, how large would the increase need to be to induce these results?

2. Comparison of Polya to Uniform Distribution with Respect to Interactions

Application of the Hughes Salvo Model using a uniform shot distribution has shown interesting interactions between pairs of parameters. It has also been shown that the Polya distribution assuming perfect battle damage assessment leads to a higher fraction of red killed for like forces. The natural extension is to carry this same investigation of pairwise comparisons to the Polya perfect information model to explore if the fractional exchange ratio skews higher in favor of blue.

3. Effect of a Standoff Capability

The Hughes Salvo Model assumes that every unit can target every other unit. What would the effect be under the uniform and Polya distributions of a force that possessed a unit with sufficient range that could engage the opponent without placing itself in range of the opponent's weapons? One would expect that blue would naturally perform better if it possessed this capability, but does it outweigh the benefit of increasing a parameter of standard blue forces parameters? Or is it simply an advantage that helps sometimes but is not crucial to the outcome of the engagement.

4. Exploration of Hughes Salvo Model with Different Distribution for Assignment of Shots

The Polya distribution was addressed as a possible alternative to shot distribution based on a behavior that shooters may follow. This of course is only one possibility of numerous distributions that can be explored. Perhaps a different distribution will provide an insight into how targets should be allocated to best defeat red forces while preserving blue survivability.

5. Comparisons using Heterogeneous and/or Asymmetric Forces

This thesis explored cases of like forces, with the exception of when blue is give advantages as indicated. However, if the United States were to engage in naval combat at present, it is most likely not going to occur with forces of comparable strength. By estimating parameters of current combatants based on their design, simulations can be developed that are more realistic in the form of tailored scenarios. For example a surface action group facing a set of gunboats equipped with modern missiles.

6. Cost Based Comparisons

Analysis conducted in this thesis does not consider the costs of granting advantages to forces. Although a combination of staying power and force size may be desirable, it is costly, requiring selecting a less attractive alternative due to budget constraints. There is obviously a hierarchy of costs, i.e., adding a ship is most likely the most expensive option, followed by perhaps increasing staying power. However, is adding a ship more cost effective than increasing firepower and defensive power? Or, is it more cost effective to build two smaller, less protected, offensive ships than a single ship that can "do it all"? Once an estimate of what it costs to add a capability is determined, and accounting for expected combat losses by choosing those capabilities, some effective cost-benefit measures can be gained.

7. Tactics versus Procurement

This thesis focuses on design, with little consideration of tactics. For the tactical commander, the decision to add a unit of staying power is moot. What he can affect is the information, force size, the training of his force, or how his subordinate units prioritize targeting. It would be interesting to take the perspective of a force commander and investigate how changing the parameters he can tactically exploit yield changes in the battle.

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